

Modified Super Efficiency in Presence of Infeasibility: A Nonradial Approach

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Abstract. As regards of supper efficiency models which increased the discrimination power of the standard DEA models, still infeasibility may occur. In literature there exists some models overcome this difficulty. In this paper a new procedure has been mooted in order to remove this shortcoming in a way that both input savings and output surplus are being considered. This procedure deals with nonradial changes for both inputs and outputs at the same time. The great feature of this model is its simplicity and that correspondence linear counter part can be easily written.

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1. Introduction

Data envelopment analysis (DEA) is a mathematical tool for relative efficiency evaluation of set of Decision Making Units (DMUs). This technique is developed by Charnes et al. (1978) and generalized by Banker et al (1984). DEA methodology on baseis of some preliminary assumptions estimates the efficient frontier. According to a comparison process to this frontier, the efficiency scores will be obtained. Those units located onto this frontier are referred to as efficiency, with the efficiency score of 1, and those far away from this curve are inefficient units, with the efficiency score less than 1.

One of the important subjects in DEA is to rank efficient units since these units are not comparable among themselves. Andersen and Petersen (1993) introduced super-efficiency DEA models for ranking efficient units. In this method by excluding each of DMUs from the reference set, super-efficiency DEA models are obtained and these scores can be used for ranking efficient DMUs. Applications of the super-efficient DEA model have increased in last few years.

One important issue in this field is infeasibility of super-efficiency DEA model. It is worthy of attention that there exist necessary and sufficient conditions for infeasibility in various super-efficiency DEA models developed by Seiford and Zhu, (1999).

Chen (2005), in his paper, has shown that, if the variable returns to scale (VRS) frontier consists of increasing, constant, and decreasing returns to scale DMUs, one of the input-oriented and output-oriented super-efficiency DEA models must be feasible. The important issue that he has considered is the use of both input- and output-oriented super-efficiency models to represent the super-efficiency completely.

It should be noted that in a paper Foroughi et al. (2006) has been presented some counterexamples and comments to the contention by Chen (2005).

As noted in chen et al. (2004), with the DEA efficiency scores super-efficiency may still contain inefficiency thus input savings or output surpluses may not be entirely represented. Therefore, they have developed a non-radial super-efficiency DEA (NRSE-DEA) approach to develop

the ranking method. Moreover they have verified that a stepwise set of proportional changes in the input (outputs) omit the slack in the radial methods.

Zhu et al. (2009) presented an approach providing super-efficiency scores which are equivalent to those of original model. They have demonstrated that for efficient DMUs, those are infeasible under the super-efficiency model, their approach yields optimal solutions and scores that show super-efficiency in both inputs and outputs.

Jahanshahloo et al. (2004) presented a method for ranking efficient DMUs in DEA models with constant and variable returns to scale. Their method is based on leave-one-out idea and l1-norm. The proposed method removes the existing difficulties in some of the existing methods in literature. The great feature of their method is that it does not suffer from infeasibility.

In a paper provide by Lee et al. (2011) the super efficiency DEA in presence of infeasibility has been dealt with which overcomes the infeasibility of super efficiency DEA models. Moreover according to what Lee et al. (2011) has been mooted, Chen and Liang (2011) provided a one model approach in order to consider infeasibility situations.

Chen et al. (2011) have proposed a modified approach for super efficiency according to simultaneous input-output projection. They have mentioned that this method shows super efficiency in both inputs and outputs at the same time thus it fully characterize the super efficiency that may exist within a DMU. The predictability of the proposed method is the key feature of it.

Researches presented some models, some of them are of two phase approach and some other is unified for overcoming infeasibility of super efficiency DEA models. Some of these models only deals with inputs, some other only considers outputs and some other considers both inputs and outputs at the same time. The presented procedure in this paper deals with inputs and outputs, both input savings and output surplus, and thus shows super efficiency that may exist in inputs and outputs or both. The important feature of the proposed model is that it considers both input savings and output surplus at the same time. Moreover its linear counterpart can be considered so simply without variable trans-

formation. Finally for purpose of clarity an applicant has also been considered.

The paper unfolds as follows: First some preliminaries about supper efficiency models will be discussed then the presented procedure for ranking units will be explained in which the existing shortcoming of infeasibility removed. Finally with a numerical example, the validity of this method will be demonstrated. Section 5 concludes the paper.

2. Preliminaries

In DEA literature ranking efficient DMUs, those referred to by the efficiency score of 1 by DEA model, has gained great deal of attention as they are not comparable among themselves. One of the famous approaches for ranking efficient DMUs is super efficiency model introduced by Andersen and Petersen (1993). This method is based on the leave-one-out idea and then comparing this left unit from the new frontier constructed through the remaining DMUs. As stated in DEA literature by super efficiency score, in the input-oriented model, a measure of the proportional increase in the inputs, for under-evaluation DMU, can be provided without destroying corresponding the efficient status while this DMU is compared with the frontier constructed through the others. The same interpretation is true for output orientation super efficiency models. According to what has been provided in literature, with input-oriented super-efficiency DEA model it is possible to measure the input super-efficiency while outputs are being considered fixed at their current levels. And, as regards of an output oriented super-efficiency DEA model, it is possible to measure the output super-efficiency while inputs are being considered fixed at their current levels.

Consider the following model which is radial supper efficiency DEA model under variable returns to scale in input orientation.

$$\begin{aligned}
& \min \quad \theta \\
& \text{s.t.} \quad \sum_{\substack{j=1 \\ j \neq o}} \lambda_j x_{ij} \leq \theta x_{ip}, \quad i = 1, \dots, m, \\
& \quad \quad \sum_{\substack{j=1 \\ j \neq o}} \lambda_j y_{rj} \geq y_{rp}, \quad r = 1, \dots, s, \\
& \quad \quad \sum_{\substack{j=1 \\ j \neq o}} \lambda_j = 1, \\
& \quad \quad \lambda_j \geq 0, \quad j = 1, \dots, n.
\end{aligned} \tag{1}$$

There may exist output surpluses which lead to infeasibility.

Now consider the following model, which is radial super efficiency DEA model in variable returns to scale in output orientation.

$$\begin{aligned}
& \max \quad \phi \\
& \text{s.t.} \quad \sum_{\substack{j=1 \\ j \neq o}} \lambda_j x_{ij} \leq x_{ip}, \quad i = 1, \dots, m, \\
& \quad \quad \sum_{\substack{j=1 \\ j \neq o}} \lambda_j y_{rj} \geq \phi y_{rp}, \quad r = 1, \dots, s, \\
& \quad \quad \sum_{\substack{j=1 \\ j \neq o}} \lambda_j = 1, \\
& \quad \quad \lambda_j \geq 0, \quad j = 1, \dots, n.
\end{aligned} \tag{2}$$

For those units which have input savings this model may also be infeasible.

3. Main Idea

According to what has been discussed in the above, in this section we present some models which has the ability to avoid infeasibility.

Let us assume that there are n DMUs to be evaluated. Each DMU is assumed to produce s different outputs via m different inputs. Let us assume that the observed **positive** input and output vectors of DMU_j are $X_j = \{x_{1j}, \dots, x_{mj}\}$ and $Y_j = \{y_{1j}, \dots, y_{sj}\}$, respectively. Consider DMU_o as an extreme efficient unit, then When (X_o, Y_o) is eliminated

from T_v , the new production possibility set T'_v has been defined as follows.

$$T'_v = \{(x, y) \mid x \geq \sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j x_j, y \leq \sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j y_j, \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, j = 1, \dots, n\}.$$

Moreover it should be noted that excluding an inefficient and weak efficient DMUs will not alter the production possibility set. Consider the following nonradial models, one is in an input orientation and the other is in an output orientation.

$$\begin{aligned} P_0 = \min \quad & \frac{1}{m} \sum_{i=1}^m \theta_i^s \\ \text{s.t.} \quad & \sum_{\substack{j=1 \\ j \neq p}} \lambda_j x_{ij} \leq \theta_i^s x_{ip}, \quad i = 1, \dots, m, \\ & \sum_{\substack{j=1 \\ j \neq p}} \lambda_j y_{rj} \geq y_{rp}, \quad r = 1, \dots, s, \\ & \sum_{\substack{j=1 \\ j \neq p}} \lambda_j = 1, \\ & \lambda_j \geq 0, \quad j = 1, \dots, n. \end{aligned} \quad (3)$$

$$\begin{aligned} P_{00} = \max \quad & \frac{1}{s} \sum_{r=1}^s \phi_r^s \\ \text{s.t.} \quad & \sum_{\substack{j=1 \\ j \neq p}} \lambda_j x_{ij} \leq x_{ip}, \quad i = 1, \dots, m, \\ & \sum_{\substack{j=1 \\ j \neq p}} \lambda_j y_{rj} \geq \phi_r^s y_{rp}, \quad r = 1, \dots, s, \\ & \sum_{\substack{j=1 \\ j \neq p}} \lambda_j = 1, \\ & \lambda_j \geq 0, \quad j = 1, \dots, n. \end{aligned} \quad (4)$$

If one wants to consider both input and output orientation, the suitable

model would be as follows which is the enhanced Russel model reference:

$$\begin{aligned}
 p_1 = \min \quad & \frac{\frac{1}{m} \sum_{i=1}^m \theta_i}{\frac{1}{s} \sum_{r=1}^s \phi_r} \\
 \text{s.t.} \quad & \sum_{\substack{j=1 \\ j \neq p}} \lambda_j x_{ij} \leq \theta_i x_{ip}, \quad i = 1, \dots, m, \\
 & \sum_{\substack{j=1 \\ j \neq p}} \lambda_j y_{rj} \geq \phi_r y_{rp}, \quad r = 1, \dots, s, \\
 & \sum_{\substack{j=1 \\ j \neq p}} \lambda_j = 1, \\
 & 0 < \theta_i \leq 1, \quad \phi_r \geq 1, \quad i = 1, \dots, m, \quad r = 1, \dots, s, \\
 & \lambda_j \geq 0, \quad j = 1, \dots, n.
 \end{aligned} \tag{5}$$

Consider the following figure, in Figure 1, the optimal solution of model (5) is strictly less than one. Consider DMU_c which is infeasible under output orientation SE-DEA model and DMU_B which is infeasible under input orientation SE-DEA model. While considering model (5), the reference point for DMU A, is \bar{A} .

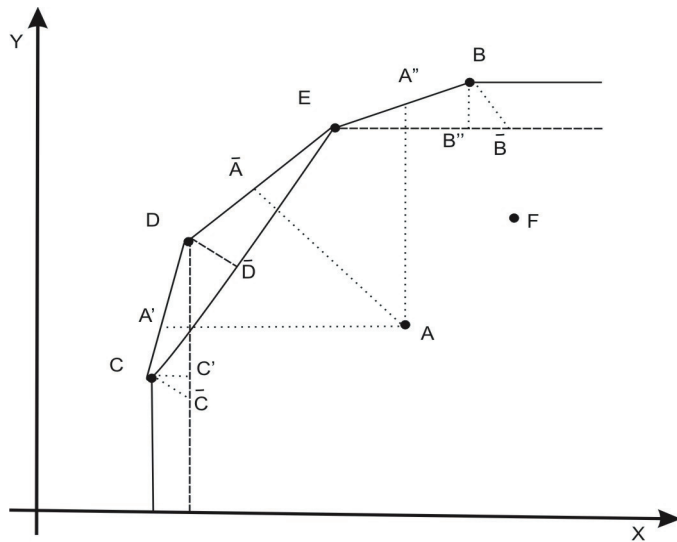


Figure 1: One input one output PPS.

Now consider the following model, which is proposed in order to overcome the occurrence of infeasibility in evaluating some DMUs. The important issue ought to be mentioned in this model is that $0 < \theta_i \leq 1$, for all i , and $0 \leq \phi_r \leq 1$, for all r .

$$\begin{aligned}
 p_2 = \min \quad & \frac{\frac{1}{m} \sum_{i=1}^m \theta_i^{sr}}{\frac{1}{s} \sum_{r=1}^s \phi_r^{sr}} \\
 \text{s.t.} \quad & \sum_{\substack{j=1 \\ j \neq p}} \lambda_j x_{ij} \leq \theta_i^{sr} x_{ip}, \quad i = 1, \dots, m, \\
 & \sum_{\substack{j=1 \\ j \neq p}} \lambda_j y_{rj} \geq \phi_r^{sr} y_{rp}, \quad r = 1, \dots, s, \\
 & \sum_{\substack{j=1 \\ j \neq p}} \lambda_j = 1, \\
 & \theta_i^{sr} \geq 1, \quad 0 < \phi_r^{sr} \leq 1, \quad i = 1, \dots, m, \quad r = 1, \dots, s, \\
 & \lambda_j \geq 0, \quad j = 1, \dots, n.
 \end{aligned} \tag{6}$$

Consider Figure 1, DMU D, while assessing with model (6), the optimal score is strictly greater than one and the reference point is \tilde{D} . For nonextreme efficient DMUs the optimal scores of models (5) and (6) are equal to one.

Model (6) is proposed for overcoming infeasibility. The great feature of this model is the nonradial essence of it which helps to find a reference point for extreme efficient units. This model let units to find their reference point with simultaneous changes in inputs and outputs.

Theorem: The optimal solution of model (5) is equals one if and only if the DMU under evaluation is non extreme efficient.

Proof. let $P_2^*=1$ thus

$$\frac{1}{m} \sum_{i=1}^m \theta_i^{sr*} = 1 \quad \text{and} \quad \frac{1}{s} \sum_{r=1}^s \phi_r^{sr*} = 1$$

Since (6) is feasible thus

$$\varphi^{sr*} \geq 1, \quad \forall r$$

and by assuming

$$\theta_i^{sr*} = \theta_i^s, \quad \forall i$$

derive a feasible solution for (3) and it is obvious that

$$\frac{1}{m} \sum_{i=1}^m \theta_i^{s*} \leq \frac{1}{m} \sum_{i=1}^m \theta_i^{sr*} \leq 1.$$

Similarly it can be concluded that (4) is also feasible and

$$\frac{1}{s} \sum_{r=1}^s \varphi_r^{s*} \geq \frac{1}{s} \sum_{r=1}^s \varphi_r^{sr*} \geq 1.$$

On the other hand if (3) is feasible and

$$0 < \frac{1}{m} \sum_{i=1}^m \theta_i^{s*} \leq 1$$

then it is possible to have a feasible solution for (6) such as:

$$\theta_i^{sr*} = \theta_i^{s*}, \quad \forall i, \quad \varphi_r^{sr*} = 1, \quad \forall r$$

thus

$$P_2^* = \frac{1}{m} \sum_{i=1}^m \theta_i^{s*}$$

moreover

$$\frac{1}{m} \sum_{i=1}^m \theta_i^{s*} \geq P_2^*.$$

As like what has been discussed, it can be concluded that if (4) is feasible and

$$\frac{1}{s} \sum_{r=1}^s \varphi_r^{s*} \geq$$

then (6) is also feasible and

$$\frac{1}{\frac{1}{s} \sum_{r=1}^s \varphi_r^{s*}} \geq P_2^*.$$

Therefore it can be said that

$$\frac{1}{s} \sum_{r=1}^s \varphi_r^{s*} = 1, \quad \frac{1}{m} \sum_{i=1}^m \theta_i^{s*} = 1$$

which, according to models (3) and (4), implies that the DMU under evaluation is non extreme efficient.

Now assume that the DMU under evaluation is non extreme efficient therefore according to models (3) and (4)

$$\frac{1}{s} \sum_{r=1}^s \varphi_r^{s*} = 1, \quad \frac{1}{m} \sum_{i=1}^m \theta_i^{s*} = 1.$$

Based on what has been discussed above if (3) is feasible and $0 < \frac{1}{m} \sum_{i=1}^m \theta_i^{s*} \leq 1$ then (6) is feasible and $\frac{1}{m} \sum_{i=1}^m \theta_i^{s*} \geq P_2^*$. Similarly if (4) is

feasible and $\frac{1}{s} \sum_{r=1}^s \varphi_r^{s*} \geq 1$ then (6) is feasible and $\frac{1}{\frac{1}{s} \sum_{r=1}^s \varphi_r^{s*}} \geq P_2^*$. Thus

according to the hypothesis (6) is feasible. Previously, it has been shown that if (6) is feasible then (3) is also feasible and

$$\frac{1}{m} \sum_{i=1}^m \theta_i^{s*} \leq \frac{1}{m} \sum_{i=1}^m \theta_i^{sr*} \leq 1$$

where $\frac{1}{m} \sum_{i=1}^m \theta_i^{s*} = 1$ thus $\frac{1}{m} \sum_{i=1}^m \theta_i^{sr*} = 1$. Equivalently; $\frac{1}{s} \sum_{r=1}^s \varphi_r^{sr*} = 1$,

thus $P_2^* = 1$. It is worthy of attention that model (6) is fractional, and

this would be a shortcoming. This model can be simply written in a linear form if the objective function is replaced with

$$\begin{aligned}
 & \frac{1}{m} \sum_{i=1}^m \theta_i - \frac{1}{s} \sum_{r=1}^s \phi_r \\
 p_3 = \max & \quad \frac{1}{m} \sum_{i=1}^m \theta_i - \frac{1}{s} \sum_{r=1}^s \phi_r \\
 \text{s.t.} & \quad \sum_{\substack{j=1 \\ j \neq p}} \lambda_j x_{ij} \leq \theta_i x_{ip}, \quad i = 1, \dots, m, \\
 & \quad \sum_{\substack{j=1 \\ j \neq p}} \lambda_j y_{rj} \geq \phi_r y_{rp}, \quad r = 1, \dots, s, \\
 & \quad \sum_{\substack{j=1 \\ j \neq p}} \lambda_j = 1, \\
 & \quad \theta_i \geq 1, \quad 0 < \phi_r \leq 1, \quad i = 1, \dots, m, \quad r = 1, \dots, s, \\
 & \quad \lambda_j \geq 0, \quad j = 1, \dots, n.
 \end{aligned} \tag{7}$$

Theorem. Model(6) is always feasible.

Proof. Let

$$\begin{aligned}
 \theta_i &= \frac{\text{Max}_j \{x_{ij}\}}{x_{io}}, \quad \forall i = 1, \dots, m, \\
 \phi_r &= \frac{\text{Min}_j \{y_{rj}\}}{y_{ro}}, \quad \forall r = 1, \dots, s.
 \end{aligned}$$

this completes the proof. According to the constrained imposed on θ_i , for all i , and ϕ_r , for all r , it is obvious that the optimal score will be greater than zero. Moreover, when $P_3 = 0$ the DMU under evaluation is extreme efficient and vice versa, and $P_3 > 0$ implies that the DMU under assessment is non extreme efficient.

Consider DMU_o which is extreme efficient, that means:

$$\frac{\frac{1}{m} \sum_{i=1}^m \theta_i^*}{\frac{1}{s} \sum_{r=1}^s \phi_r^*} > 1$$

thus it can be concluded that;

$$\frac{\frac{1}{m} \sum_{i=1}^m \theta_i^*}{\frac{1}{s} \sum_{r=1}^s \phi_r^*} = \frac{\alpha}{\beta} > 1$$

so we define a feasible solution for (7) as

$$\frac{1}{m} \sum_{i=1}^m \theta_i^* = \alpha, \frac{1}{s} \sum_{r=1}^s \phi_r^* = \beta$$

thus

$$\frac{1}{m} \sum_{i=1}^m \theta_i - \frac{1}{s} \sum_{r=1}^s \phi_r = \alpha - \beta > 0.$$

Moreover if

$$\frac{\frac{1}{m} \sum_{i=1}^m \theta_i^*}{\frac{1}{s} \sum_{r=1}^s \phi_r^*} = \frac{\alpha}{\beta} = 1$$

so we define a feasible solution for (7) as

$$\frac{1}{m} \sum_{i=1}^m \theta_i^* = \alpha, \frac{1}{s} \sum_{r=1}^s \phi_r^* = \beta$$

therefore;

$$\frac{1}{m} \sum_{i=1}^m \theta_i - \frac{1}{s} \sum_{r=1}^s \phi_r = \alpha - \beta = 0.$$

Thus;

Theorem. There is a clear relation between these two models.

1- $P_1 = 1$ if and only if $P_2 = 0$.

2- $P_1 < 1$ if and only if $P_2 > 0$.

According to the obtained θ_i^* , φ_r^* for all i and r , stability region for DMU_p , DMU under evaluation, define as follows:

$$A = \{(X, Y) | x_{ip} \leq x_i \leq \theta_i^* x_{ip}, \quad \forall i, \varphi_r^* y_{rp} \leq y_r \leq y_{rp}, \quad \forall r\}$$

adding each member of this set like (\bar{X}, \bar{Y}) to the production possibility set of the remaining DMUs, this unit will be efficient.

Theorem. If unit (\bar{X}, \bar{Y}) belong to A and

$$\bar{x}_1 \neq \theta_1^* x_{1p} \vee \dots \vee \bar{x}_m \neq \theta_m^* x_{mp} \vee \bar{y}_1 \neq \varphi_1^* Y_{1p} \vee \dots \vee \bar{y}_s \neq \varphi_s^* Y_{sp}$$

this unit will be nonextreme efficient.

Proof. With out losing generality and for simplicity let

$$\bar{x}_1 < \theta_1^* x_{1p}, \dots, \bar{x}_m = \theta_m^* x_{mp}, \quad \bar{y}_1 = \varphi_1^* Y_{1p}, \bar{y}_s = \varphi_s^* Y_{sp}$$

therefore for evaluating unit (\bar{X}, \bar{Y}) we will have:

$$\frac{\frac{1}{m} \sum_{i=1}^m \bar{\theta}_i}{\frac{1}{s} \sum_{r=1}^s \bar{\varphi}_r} > \frac{\frac{1}{m} \sum_{i=1}^m \theta_i^*}{\frac{1}{s} \sum_{r=1}^s \varphi_r^*} \geq 1$$

Now according to Theorem (3.) this unit will be extreme efficient.

4. Application

In an example it has been shown that considering model(6) and its linear counterpart it is possible to avoid infeasibility. Thus according to the obtained result by imposing a secondary goal the DMUs can be ranked. The input-output data are gathered in Table 1, these data are from the application used in Chen et al. (2011). Considering these data the results are listed in Table 2.

Table.1 *Inputs and Outputs*

| DMU # | O1 | O2 | O3 | O4 | O5 | I1 | I2 | I3 |
|-------|-------|------|------|----------|----------|------|------|----|
| 1 | 34740 | 80.9 | 67.4 | 67.43474 | 203680.9 | 5.1 | 3 | 6 |
| 2 | 37117 | 79.4 | 63.8 | 63.8 | 1968 | 7.2 | 1.8 | 5 |
| 3 | 35712 | 78.8 | 57.3 | 57.3 | 2081 | 12.1 | 1.6 | 6 |
| 4 | 35133 | 80.3 | 72.8 | 72.8 | 2312 | 6.8 | 2.2 | 6 |
| 5 | 12152 | 75.9 | 64 | 64 | 930 | 8.9 | 1.8 | 5 |
| 6 | 47984 | 77.9 | 74.2 | 74.2 | 2133 | 5.6 | 2.4 | 4 |
| 7 | 37023 | 79 | 69.3 | 69.3 | 1461 | 2.8 | 1.6 | 6 |
| 8 | 37504 | 78.9 | 72.8 | 72.8 | 1502 | 8.4 | 1.7 | 4 |
| 9 | 33918 | 80.2 | 62.4 | 62.4 | 2055 | 9.1 | 1.9 | 4 |
| 10 | 33854 | 79.1 | 67.4 | 67.4 | 2424 | 9.2 | 2.3 | 5 |
| 11 | 20327 | 78.9 | 56 | 56 | 1167 | 9.9 | 4.6 | 5 |
| 12 | 10814 | 72.9 | 53.2 | 53.5 | 705 | 7.2 | 5.3 | 8 |
| 13 | 52764 | 81.5 | 82.9 | 82.9 | 2103 | 1.8 | 4.8 | 4 |
| 14 | 48604 | 78.4 | 62.2 | 62.2 | 1436 | 4.3 | 4.7 | 6 |
| 15 | 30200 | 80.3 | 50.1 | 50.1 | 1783 | 7.7 | 2.5 | 6 |
| 16 | 35575 | 82.3 | 60.5 | 60.5 | 1822 | 4.4 | 1 | 4 |
| 17 | 80288 | 78.4 | 55.7 | 55.7 | 2215 | 4.2 | 1.1 | 5 |
| 18 | 7298 | 75.6 | 42.6 | 42.6 | 356 | 3.6 | 5 | 25 |
| 19 | 26464 | 79.8 | 71.2 | 71.2 | 1424 | 3.7 | 2.7 | 6 |
| 20 | 38618 | 79.2 | 69.5 | 69.5 | 2070 | 4.3 | 3.5 | 5 |
| 21 | 64193 | 79.8 | 77.3 | 77.3 | 2330 | 3.5 | 1.3 | 4 |
| 22 | 7946 | 75.2 | 57.6 | 57.6 | 496 | 18.2 | 1.9 | 9 |
| 23 | 17456 | 77.7 | 67.8 | 67.8 | 1237 | 7.6 | 3.5 | 6 |
| 24 | 16308 | 79 | 49.9 | 49.9 | 730 | 3.7 | 2.8 | 5 |
| 25 | 8775 | 74.2 | 62.4 | 62.4 | 930 | 11.7 | 3.3 | 8 |
| 26 | 27226 | 80.5 | 57.2 | 57.2 | 1218 | 9.2 | 3.1 | 5 |
| 27 | 39694 | 80.5 | 74.9 | 74.9 | 1746 | 5.8 | 2.2 | 3 |
| 28 | 50532 | 81.3 | 75.3 | 75.3 | 2794 | 3.8 | 0.9 | 3 |
| 29 | 5816 | 71.4 | 26.5 | 26.5 | 255 | 10.3 | 13.7 | 38 |
| 30 | 42000 | 77.9 | 70.1 | 70.1 | 4178 | 5.1 | 1.6 | 7 |

In the following table E.1, E.2, E.3 and E.4 are stands for the optimal objective value of models (3), (4), (6) and (7).

Table.2 *Efficiencies*

| DMU # | E.1 | E.2 | E.3 | E.4 | DMU # | E.1 | E.2 | E.3 | E.4 |
|-------|-------------|-------------|-------|-------|-------|-------------|-------------|-------|-------|
| 1 | 0.515 | 1.259 | 1 | 0 | 16 | <i>inf.</i> | 1.295 | 1.002 | 0.002 |
| 2 | 0.543 | 1.274 | 1 | 0 | 17 | <i>inf.</i> | 1.143 | 1.061 | 0.057 |
| 3 | 0.459 | 1.326 | 1 | 0 | 18 | 0.446 | 4.037 | 1 | 0 |
| 4 | 0.489 | 1.19 | 1 | 0 | 19 | 0.636 | 1.452 | 1 | 0 |
| 5 | 0.509 | 2.352 | 1 | 0 | 20 | 0.58 | 1.21 | 1 | 0 |
| 6 | 0.601 | 1.108 | 1 | 0 | 21 | <i>inf.</i> | 0.961 | 1.06 | 0.057 |
| 7 | <i>inf.</i> | <i>inf.</i> | 1.066 | 0.066 | 22 | 0.339 | 3.509 | 1 | 0 |
| 8 | 0.577 | 1.287 | 1 | 0 | 23 | 0.419 | 1.808 | 1 | 0 |
| 9 | 0.547 | 1.3 | 1 | 0 | 24 | 0.667 | 2.274 | 1 | 0 |
| 10 | 0.468 | 1.232 | 1 | 0 | 25 | 0.324 | 2.875 | 1 | 0 |
| 11 | 0.393 | 1.799 | 1 | 0 | 26 | 0.434 | 1.621 | 1 | 0 |
| 12 | 0.358 | 2.746 | 1 | 0 | 27 | 0.688 | 1.179 | 1 | 0 |
| 13 | <i>inf.</i> | <i>inf.</i> | 1.357 | 0.346 | 28 | <i>inf.</i> | <i>inf.</i> | 1.299 | 0.287 |
| 14 | 0.525 | 1.362 | 1 | 0 | 29 | 0.171 | 5.997 | 1 | 0 |
| 15 | 0.451 | 1.502 | 1 | 0 | 30 | <i>inf.</i> | 1.063 | 1.071 | 0.066 |

As it can be seen, considering the proposed model, infeasibility has been avoided. It also can be seen that where ever the optimal solution of model (6) is equal to one, the optimal solution of model (7) is equals zero. In accordance to the obtained results, listed in the following table,

it comes to mind that by imposing a secondary goal, if ranking is the final aim, DMUs can be ranked.

5. Conclusion

In the current paper the relationship between super-efficiency and the infeasibility of super-efficiency DEA model has been discussed. As mentioned in previous papers in this field, for entirely presenting the super-efficiency, both input-oriented and output-oriented super-efficiency DEA models should be considered. Considering the existing super efficiency models which overcomes the infeasibility problem, this paper considers a modified approach in which the alteration of inputs and outputs has been considered at the same time. The simplicity of this approach and that its linear counter part can be easily, are the key features of this method. This procedure consider nonradial alteration of indexes in order to advantage of nonradial DEA models.

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